Massive fields temper anomaly-induced inflation: the clue to graceful exit?

Ilya L. Shapiro^{a,b,1} , **Joan Solà**^{c,d,2}

^{a} Departamento de Física Teorica, Universidad de Zaragoza,
50009, Zaragoza, Spain

^{b} Departamento de Física - Instituto de Ciencias Exatas
Universidade Federal de Juiz de Fora, 36036-330, MG, Brazil

^{c} Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona,
Diagonal 647, E-08028, Barcelona, Catalonia, Spain

^{d} Institut de Física d'Altes Energies, Universitat Autònoma de Barcelona,
08193 Bellaterra, Barcelona, Catalonia, Spain

ABSTRACT

A method of calculating the vacuum effective action for massive quantum fields in curved space-time is outlined. Our approach is based on the conformal representation of the fields action and on the integration of the corresponding conformal anomaly. As a relevant cosmological application, we find that if taking the masses of the fields into account, then the anomaly-induced inflation automatically slows down. The only relevant massive fields for this purpose turn out to be the fermion fields. So in supersymmetric theories this mechanism can be specially efficient, for it may naturally provide the graceful exit from the inflationary to the FLRW phase. Taking the SUSY breaking into account, the anomaly-induced inflation could be free of the well-known difficulties with the initial data and also with the amplitude of the gravitational waves.

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^{1}E-mail: shapiro@dftuz.unizar.es. On leave from Tomsk State Pedagogical University, Tomsk, Russia.
^{2}E-mail: sola@ifae.es.

1. Introduction

Inflation proved very useful in solving numerous problems of the theory of the Early Universe. The conventional approach, which is based on the inflaton field, is extremely helpful for the inflationary phenomenology [1]. At the same time, the origin of the inflaton remains unclear, and it is quite reasonable to look for other approaches yielding similar phenomenological results. Recently, there was an increasing interest for the cosmological applications of quantum field theory in curved space-time, specially in connection to vacuum effects [2].

An exact calculation of the vacuum effective action is possible only for some special models, usually for massless conformal invariant matter fields and special restrictions of the classical background space-time. An important example is the $d = 4$ theory on the conformally flat (or similar FLRW with $k = \pm 1$) metric, where the Reigert-Fradkin-Tseytlin effective action [3] is exact and provides the most natural theoretical background for inflation [4, 5, 6].

The anomaly-induced action can be applied at high energies, where the masses of the fields are negligible [7, 5, 6]. If, at the high-energy region, there is a supersymmetry (SUSY), one meets the stable version of the anomaly-induced inflation, which starts without any fine tuning and leads to sufficient expansion of the Universe [8]. In the course of inflation, the typical energy scale decreases, and one encounters the non-stable version (Starobinsky model [4]), which provides fast exit to the FLRW phase [5]. The transition from stable to non-stable inflation can be achieved through soft SUSY breaking and the decoupling of the massive sparticles at low energy [9]. The potential importance of supersymmetry is due to the following circumstances. The anomaly-induced inflation may be stable or unstable – depending on the particle content of the underlying quantum field theory. This opens the possibility to interpolate, in a natural way, between the stable regime at the beginning of inflation and the unstable regime at the end of inflation. In particular, the supersymmetric gauge theory may have a particle content corresponding to the stable inflation. The advantage of stable inflation is that it starts independent of the initial data for the conformal factor $a(t)$ of the metric, which can emerge after the string phase transition. [3]. If, in the course of inflation, the typical gravitational energy scale μ decreases, and if the sparticles have much bigger masses than the other particles, then they decouple and at the last stage of inflation the number of active degrees of freedom diminishes. This is possible because μ can be sensibly defined from the value of the Hubble parameter H [10], which indeed lessens during the tempered expansion caused by massive fields, as will be shown below. Therefore this mechanism automatically brings inflation into an unstable phase, with the possibility of an eventual transition to the FLRW regime.

Thus, supersymmetry and its breaking may provide a natural qualitative mechanism for the graceful exit from the inflationary phase, without fine-tuning of the parameters of the theory. Furthermore, the spectrum of the gravitational waves in this model [11, 8] can be in agreement with the existing CMBR data. Still, in this picture there is an unclear point, namely the use of the anomaly-induced action for the massive fields is not completely justified. Also, the transition to the FLRW phase may not necessary occur after the exit from the exponential inflation. The behavior of the Universe depends on the initial deviation from the exponential expansion law [7, 12, 5, 6]. It was established that the universe goes to the FLRW regime if this deviation leads to an expansion slower than exponential, while it goes to the uncontrolled "hyperinflation" [6] if it leads to a faster expansion. Therefore, it would be very nice to learn that the masses of the fields really slow down inflation. If this would be so, the graceful exit to the FLRW phase would not require any

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[3]We remark that this mechanism can not explain the homogeneity and isotropy of the initial state. Perhaps this problem can be solved only in the framework of the string-inspired inflation.

suppositions concerning initial perturbations at the instant when SUSY breaks down!

In this letter we are going to address both mentioned problems. We shall develop a simple but reliable Ansatz for the effective action of massive fields. Our approach to the derivation of the effective action is based on the Cosmon Model, which was developed in [13] for other purposes– see also [14]. The idea is to construct the conformal invariant formulation [13] of the gauge theory (Standard Model or extensions thereof, including GUT's), and then use the well-known methods to derive the anomaly-induced action. The procedure of "conformization" is known for a long time as applied to General Relativity [15] and Particle Physics [16]. At the classical level, the theory which results from this procedure is always equivalent to the original theory. Nevertheless, in the quantum theory the equivalence will be destroyed by the anomaly, which can be calculated explicitly. Besides the anomalous terms, there are the conformal invariant quantum corrections to the classical vacuum action. However, the complete method of deriving these contributions is not known, just because the effective action can not be calculated exactly for the massive $d = 4$ theories. The idea of our Ansatz is to disregard these contributions because they are, indeed, of higher order with respect to the leading ones we take into account. As we shall see, our results are in perfect agreement with the renormalization group. This provides better understanding of the applicability of our approach.

2. Conformization and effective action

Our first purpose is to construct such a formulation of the Standard Model (SM) in curved space-time which possesses local conformal invariance in $d = 4$. Actually, the procedure can be applied to any gauge theory and we are especially aiming at a realistic supersymmetric gauge theory, providing stability for the anomaly-induced inflation.

The original action of the theory includes kinetic terms for spinor and gauge boson fields, as well as interaction terms, all of them already conformal invariant. As for scalars (e.g. Higgs bosons) we suppose that their kinetic terms appear in the combination $g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + 1/6 \cdot R\varphi^2$ providing the local conformal invariance. The non-invariant terms are the massive ones for the scalar and spinor fields:

$$\frac{1}{2}\int d^4x\sqrt{-g}\, m_H^2\,\varphi^2\,, \qquad \int d^4x\sqrt{-g}\, m\,\bar{\psi}\psi\,. \tag{1}$$

Furthermore, there is an action for gravity itself, which is also non-invariant

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\, R\,. \tag{2}$$

In all mentioned cases the conformal noninvariance is caused by the presence of dimensional parameters m_H^2, m, $M_P^2 = 1/G$. The central idea of the Cosmon Model [13] is to replace these parameters by functions of some new auxiliary scalar field χ. For instance, we replace

$$m_H^2 \rightarrow \frac{m_H^2}{M^2}\,\chi^2\,, \qquad m \rightarrow \frac{m}{M}\,\chi\,, \qquad M_P^2 \rightarrow \frac{M_P^2}{M^2}\,\chi^2\,, \tag{3}$$

where M is some dimensional parameter, e.g. related to a high scale of spontaneous breaking of dilatation symmetry [16]. It is supposed that the new scalar field χ takes the values close to M, especially at low energies. But, there is a great difference between χ and M with respect to the conformal transformation. The mass does not transform, while χ does. Then, the action of the new model becomes invariant under the conformal transformation

$$\chi \rightarrow \chi\, e^{-\sigma}\,, \tag{4}$$

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which is performed together with the usual transformations for the other fields

$$g_{\mu\nu} \to g_{\mu\nu}\, e^{2\sigma}\,, \quad \varphi \to \varphi\, e^{-\sigma}\,, \quad \psi \to \psi\, e^{-3/2\,\sigma}\,. \tag{5}$$

It is easy to see that, in the matter field sector, the terms (1) are replaced, under (3), by Yukawa and quartic scalar interaction terms. These interactions are between physical fields (spinors and scalars) and the new auxiliary scalar χ. Thus, in the matter sector our program of "conformization" is complete.

However, in the gravity sector (2) the conformal symmetry holds only for $\sigma = const$, i.e. only at the level of global dilatation symmetry, and this is still not what we need. Let us make one more step and require the local conformal invariance. Then the gravity action must be replaced by the expression [15]

$$S^*_{EH} = -\frac{1}{16\pi G\, M^2} \int d^4x \sqrt{-g}\, \left[\, R\chi^2 + 6\,(\partial\chi)^2\,\right]\,, \tag{6}$$

where $(\partial\chi)^2 = g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$. After setting $\chi \to M$ the expression (6) becomes identical to the initial one (2). This fixing can be called "conformal unitary gauge" in analogy with the unitary gauge of ordinary gauge theories, and the scale M can be associated to the vacuum expectation value of the spontaneously broken dilatation symmetry at high energies [16, 13, 14]. But, as far as we consider the space-time dependence of χ and define its conformal transformation, the resulting theory exhibits local conformal invariance under (4) and (5) with $\sigma = \sigma(x)$. The new conformal symmetry is introduced simultaneously with the new scalar field χ, which absorbs the degree of freedom of the conformal factor of the metric. The new theory satisfies, at the classical level, the conformal Noether identity

$$\left[\, 2\,\frac{g_{\mu\nu}}{\sqrt{-g}}\,\frac{\delta}{\delta g_{\mu\nu}} - \frac{\chi}{\sqrt{-g}}\,\frac{\delta}{\delta\chi} + d_{\Phi_i}\,\frac{\Phi_i}{\sqrt{-g}}\,\frac{\delta}{\delta\Phi_i}\,\right] S^c_t = 0\,, \tag{7}$$

where Φ_i stands for the matter fields of different spins, d_{Φ_i} denote their conformal weights and $S^c_t = S^c_t[g_{\mu\nu}, \chi, \Phi_i]$ is the total (classical) action including the modified gravitational term (6).

When we quantize the theory, it is important to separate the quantum fields from the ones which represent a classical background. In order to maintain the correspondence with the usual formulation of the SM, we avoid the quantization of the field χ which will be considered, along with the metric, as an external classical background for the quantum matter fields. It is well known (see, e.g. [17]) that the renormalizability of the quantum field theory in external fields requires some extra terms in the classical action of the theory. The list of such terms includes the nonminimal term of the $\int R\varphi^2$-type in the Higgs sector, and the action of external fields with the proper dimension and symmetries. The higher derivative part of the vacuum action has the form

$$S_{vac} = \int d^4x \sqrt{-g}\, \left\{l_1 C^2 + l_2 E + l_3 \nabla^2 R\right\}\,, \tag{8}$$

where, $l_{1,2,3}$ are some parameters, C^2 is the square of the Weyl tensor and E is the integrand of the Gauss-Bonnet topological invariant. Now, since there is an extra field χ, the vacuum action should be supplemented by the χ-dependent term. The only possible, conformal and diffeomorphism invariant, terms with dimension 4 are (6) and the $\int \chi^4$-term. The last contributes to the cosmological constant, which we suppose to cancel and do not consider here in order to keep the discussion clear and compact. The effect of the cosmological constant will be reported elsewhere.

The next step is to derive the conformal anomaly in the theory with two background fields $g_{\mu\nu}$ and χ. Here we follow the strategy used in a similar situation [18]. The anomaly results from the renormalization of the vacuum action [19] including the terms (6) and (8). For the sake of generality, let us suppose that there is also some background gauge field with strength tensor $F_{\mu\nu}$. Then the conformal anomaly has the form

$$< T_\mu^\mu > = -\left\{ wC^2 + bE + c\nabla^2 R + dF^2 + f\left[R\chi^2 + 6\,(\partial\chi)^2\right]\right\}, \qquad (9)$$

where w, b, c are the β-functions for the parameters l_1, l_2, l_3, and f is the β-function for the dimensionless parameter $1/(16\pi G\,M^2)$ of the action (6) which will play an essential role in our considerations. Finally, d is the β-function for the gauge coupling constant, which is standard. The values of w, b and c depend on the particle content of the model and are the following (see e.g. [6])

$$w = \frac{N_0 + 6N_{1/2} + 12N_1}{120 \cdot (4\pi)^2}\,, \quad b = -\frac{N_0 + 11N_{1/2} + 62N_1}{360 \cdot (4\pi)^2}\,, \quad c = \frac{N_0 + 6N_{1/2} - 18N_1}{180 \cdot (4\pi)^2}\,. \qquad (10)$$

Recall that the condition for stable inflation is $c > 0$ [4]. Then one can play with various models; e.g. from the previous equation it follows that the particle content of the SM ($N_0 = 4, N_{1/2} = 24, N_1 = 12$) leads to $c < 0$ (unstable inflation) whereas for the Minimal Supersymmetric Standard Model (MSSM) [20] ($N_0 = 104, N_{1/2} = 32, N_1 = 12$) one has $c > 0$ (stable inflation) etc. On the other hand from direct calculation using the Schwinger-DeWitt method (see e.g. [17] and references therein) we get

$$f = \sum_i \frac{N_i}{3\,(4\pi)^2} \frac{m_i^2}{M^2}\,, \qquad (11)$$

where N_i are the number of Dirac spinors with masses m_i. We note that bosons do not contribute to f.

In order to obtain the anomaly-induced effective action, we put $g_{\mu\nu} = \bar{g}_{\mu\nu}\cdot e^{2\sigma}$ and $\chi = \bar{\chi}\cdot e^{-\sigma}$, where the metric $\bar{g}_{\mu\nu}$ has fixed determinant and the field $\bar{\chi}$ does not change under the conformal transformation. Then, the solution of the equation for the effective action $\bar{\Gamma}$ proceeds in the usual way [3, 17, 18]. Disregarding the conformal invariant term [17] we arrive at the following expression:

$$\bar{\Gamma} = \int d^4x\sqrt{-\bar{g}}\,\{w\bar{C}^2 + b(\bar{E} - \frac{2}{3}\bar{\nabla}^2\bar{R}) + 2b\,\sigma\bar{\Delta} + d\bar{F}^2 +$$

$$+ f\left[\bar{R}\bar{\chi}^2 + 6\,(\partial\bar{\chi})^2\right]\}\sigma - \frac{3c + 2b}{36}\int d^4x\sqrt{-g}\,R^2\,, \qquad (12)$$

which is the quantum correction to the classical action of vacuum.

Let us compare Eq.(12) with the quantum correction from the renormalization group. The expansion of the homogeneous, isotropic universe means a conformal transformation of the metric $g_{\mu\nu}(t) = a^2(\eta)\,\bar{g}_{\mu\nu}$, where $a(\eta) = \exp\sigma(\eta)$ and η is the conformal time. On the other hand, the renormalization group in curved space-time corresponds to the scale transformation of the metric $g_{\mu\nu} \to g_{\mu\nu}\cdot e^{-2t}$ simultaneous with the inverse transformation of all dimensional quantities [21, 17]. For any μ we have $\mu \to \mu\cdot e^t$. Thus, one can compare the dependence of the anomaly-induced effective action (12) on σ and the scale dependence of the renormalization-group improved

classical action. The last is defined through the solution of the renormalization group equation for the effective action [21, 17]

$$\Gamma[e^{-2t}g_{\alpha\beta}, \Phi_i, P, \mu] = \Gamma[g_{\alpha\beta}, \Phi_i(t), P(t), \mu]\,, \tag{13}$$

where Φ_i is, as before, the set of all fields and P the set of all parameters of the theory. In the leading-log approximation one can take, instead of (13), the classical action and replace (for the massless conformal theory) $P \to P_0 + \beta_P\, t$. Now, comparing (12) with the result of this procedure, one confirms the complete equivalence of the two expressions in the terms which do not vanish for $\sigma = const$. In particular, coefficient f is a factor of the β-function for the Newton constant G. The important general conclusion is that the anomaly-induced effective action is a direct generalization of the renormalization group improved classical action. On the other hand, the correspondence in the f-term justifies the correctness of our approach and also helps to learn the limits of its validity.

3. The role of masses in tempering inflation

In order to understand the role of the particle masses in the anomaly-induced inflation, let us consider the total action with quantum corrections

$$S_t = S_{matter} + S_{EH} + S_{vac} + \bar{\Gamma}\,, \tag{14}$$

which does not satisfy the Noether identity (7) because of the conformal anomaly. We notice that the account of quantum corrections into the matter sector would be senseless, because matter and radiation can be treated incoherently as a fluid. The only important features of the matter action are the energy density ρ, pressure p and their dependence on a. Of course, quantum effects may change these dependencies, but we can always choose some model for $p(a)$ and $\rho(a)$ without going into the details of quantum effects. On the other hand, as far as we suppose $\rho \ll M_P^4$ during the inflation period, the matter-radiation content can not really affect the expansion of the Universe. Since $a(\eta)$ grows very fast during inflation, the energy density greatly decreases in a very short time and can not play any role. Concerning pressure, its importance is even smaller, because matter is out of equilibrium during the inflation.

One of the approximations we made was to disregard higher loop and non-perturbative effects in the vacuum sector. There is an attractive possibility to consider the strong interacting regime using the AdS/CFT correspondence [22, 23], but this goes beyond the scope of the present letter. Another approximation is that we take (as the comparison with the renormalization group shows) only the leading-log corrections. Usually, this is justified if the process goes at high energy scale. If the quantum theory has UV asymptotic freedom, the higher loops effects are suppressed, and our approximation is reliable. At the low-energy limit, we suppose that the massive fields decouple and their contributions are not important. Then Eq. (14) can be presented in the form

$$S_t = \int d^4x \sqrt{-\bar{g}} \left\{ \left(-\frac{M_P^2}{16\pi M^2} + f\sigma \right) [\bar{R}\bar{\chi}^2 + 6\,(\partial\bar{\chi})^2\,]\right.$$
$$\left. - \left(\frac{1}{4} - d\sigma\right) \bar{F}^2 \right\} + S_{matter} + high.\,deriv.\,terms\,. \tag{15}$$

One can see that the modifications with respect to the case of free massless fields [6] are an additional f-term and the contribution to anomaly due to the background gauge fields.

In order to restore the Hilbert-Einstein term and get the inflationary solution, we fix the conformal unitary gauge and put $\chi = \bar{\chi} e^{\sigma} = M$. Furthermore, we can choose the conformally flat metric $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$. Then the gravitational part of the action (15) becomes

$$S_{grav} = \int d^4x \left\{ 2b \left(\partial^2 \sigma \right)^2 - (3c + 2b) \left[(\partial\sigma)^2 + \partial^2\sigma \right]^2 - \right.$$

$$\left. - 6M_P^2 e^{2\sigma} (\partial\sigma)^2 \left[1 - \frac{16\pi M^2}{M_P^2} f \right] - \left(\frac{1}{4} - d\sigma \right) \bar{F}^2 \right\}. \tag{16}$$

Computing the equation of motion in terms of the physical time t (where $dt = a(\eta)d\eta$) we find

$$a^2 \dddot{a} + 3a\dot{a}\ddot{a} - \left(5 + \frac{4b}{c} \right) \dot{a}^2 \ddot{a} + a\ddot{a}^2 - \frac{M_P^2}{8\pi c} \left(a^2\ddot{a} + a\dot{a}^2 \right) +$$

$$+ \frac{2fM^2}{c} \ln a \left(a^2\ddot{a} + a\dot{a}^2 \right) + \frac{2fM^2}{c} \frac{\dot{a}^2}{a} - \frac{d\bar{F}^2}{6ca} = 0. \tag{17}$$

An exact solution of this fourth order non-linear differential equation does not look possible, but it can be easily analyzed within the approximation that f is not too large. Then the new terms (collected in the second line of Eq. (17)) can be considered as perturbations. Moreover, the last two of them are irrelevant, because during inflation they decrease exponentially with respect to the other terms. Thus, in this approximation, the only one relevant change is the replacement

$$M_P^2 \longrightarrow M_P^2 \left[1 - \tilde{f} \ln a(t) \right], \tag{18}$$

where for future convenience we have introduced the dimensionless parameter

$$\tilde{f} \equiv \frac{16\pi f M^2}{M_P^2} = \sum_i \frac{N_i}{3\pi} \frac{m_i^2}{M_P^2}. \tag{19}$$

Notice that f is given by Eq. (11) and so $\tilde{f}$ does not depend on the scale M. Since (18) is a slowly varying function, the effect of the masses may be approximated through the modification of the inflation law

$$a(t) = a_0 e^{H_1 t}, \qquad H_1 = const \tag{20}$$

according to [4]

$$H_1 = \frac{M_P}{\sqrt{-16\pi b}} \longrightarrow \frac{M_P}{\sqrt{-16\pi b}} \left[1 - \tilde{f} \ln a(t) \right]^{1/2} = H(t), \tag{21}$$

To substantiate our claim, we have solved Eq. (17) directly using the numerical methods. The plots corresponding to the numerical solution of the Eq. (17) using Mathematica [24] are shown in Fig. 1. Since in the first period of inflation masses do not play much role and the stabilization of the exponential inflation proceeds very fast [6], the initial data (in both Eq. (21) and the plots of Fig. 1) were chosen according to the exponential inflation law:

$$a(0) = 1, \qquad \dot{a}(0) = H_1, \qquad \ddot{a}(0) = H_1^2, \qquad \dddot{a}(0) = H_1^3. \tag{22}$$
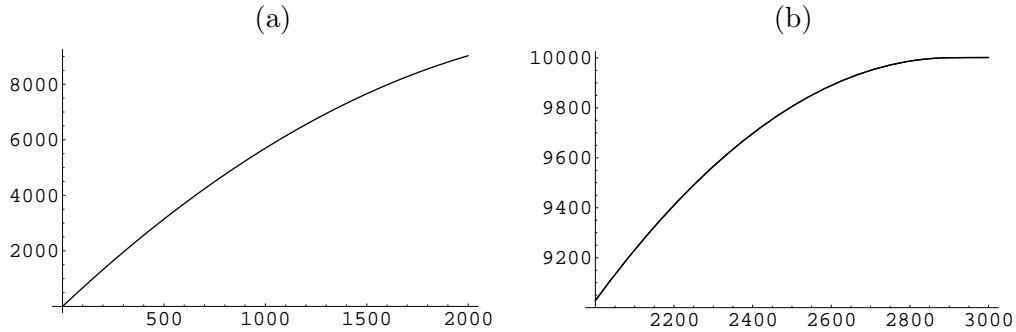
Figure 1. *(a) Plot of $\ln(a)$ versus the physical time t as a result of the numerical analysis of Eq.(17); t is given in units of $16\pi/M_P$ and we fixed the parameter (19) as $\tilde{f} = 10^{-4}$. In this time interval, inflation does not stop, yet; (b) As in (a), but extending the numerical analysis until reaching an approximate plateau marking the end of stable inflation.*

According to the numerical analysis, the total number of e-folds in the "fast phase" of inflation (until the Hubble constant becomes comparable to the SUSY breaking scale) is about 10^4 for our particular values of the parameters, and at the last stage the expansion essentially slows down. The chosen value of the parameter (19) $\tilde{f} = 10^{-4}$ in the plot is, as we warned before, independent of the scale M, and it determines where the process of stable inflation finishes as well as the number of e-folds. Following the above considerations, the transition from the stable to the unstable inflation can be associated to a high energy scale which we shall call M^*. Let us remark that this typical scale M^* may be quite different from the scale of SUSY breaking M_{SUSY}, in particular M^* can be some orders of magnitude below M_{SUSY}. The scale M^* is such that there is a sufficient number of sparticles (scalars and fermions) lighter than M^*, so that $c > 0$ even well below $M_{SUSY}-$ see eq.(10). As an illustration, let us indicate the unique example of the gauge theory where the spectrum of masses is known: the Standard Model of particle physics. In the SM the symmetry breaking scale is given by the vacuum expectation value, v, of the Higgs field, from which one defines the Fermi scale $M_F = G_F^{-1/2} \approx 300\,GeV$. However most of the particles have masses much below M_F and v, and even below $1\,GeV$. One can suppose that a similar situation takes place in the high energy SUSY GUT. As we are going to discuss below, the constraint $M^* < 10^{14}\,GeV$ provides better properties of the metric perturbations. It is important that this does not put rigid limitations on the value of the scale of supersymmetry breaking which can be some orders of magnitude greater that M^*.

One has to notice that the scale M_{SUSY} and corresponding M^* are not necessarily linked to a high energy SUSY scale (e.g. SUSY-GUT, $M_X \sim 10^{16}\,GeV$) but it could just be the SUSY breaking scale of the MSSM at the TeV scale [20]. In the last case, however, the total number of inflation e-folds would be much greater, but this would not lead to any qualitative change on the shape of the plots of Fig. 1 as can be seen from the analytical structure of eqs.(17)-(19) [5].

The important qualitative point is that for any value of $\tilde{f}$ the approximate plateau eventually appears and signals the end of stable inflation. Also notice from Fig. 1 that the initial evolution is close to the exponential inflation (20), but after that the expansion slows down due to the quantum

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[4]We remind the reader that the coefficient b is negative for any particle content, see eq.(10).

[5]In Fig. 1 we have just illustrated a situation where the numerical analysis is sufficiently simple, corresponding to f not too small and so based on a SUSY-GUT scale. For smaller and smaller $\tilde{f}$ the computer time becomes exceedingly long.

effects of massive fermions. The general behaviour is close to formula (21). According to the plot in Fig.1 (b), the evolution tends to $H = 0$, but before this there must be a breaking of SUSY and the transition to the unstable phase. Qualitatively and quantitatively, the plot is in a good correspondence with formula (21), especially in the region $\tilde{f} \ln a(t) \ll 1$ where it can be safely used to simplify the analysis. Remember that the effective action (12) has been derived in the leading-log approximation, such that the effect of particles masses has been reduced to the renormalization of the Newton constant. Indeed, this approximation is valid only at high energies, when $H \gg m_i$ for all the fermions. Also, formula (21) is based on treating the $\tilde{f} \ln a(t)$-term as a small perturbation.

4. Graceful exit from anomaly-induced inflation

In order to solve the graceful exit problem in our framework we do not need to insist that the rate of change of the scale factor, $H(t)$, reduces to zero at some point. Recall that $H(t)$ sets the scale μ of the renormalization group running for the gravitational part. If we consider the SUSY breaking and the corresponding change in the number of active degrees of freedom [9], then the necessary and sufficient condition for the applicability of our approach is that $H(t)$ decreases from the initial value about [6] $M_P/\sqrt{-16\pi b} \sim 10^{18}\,GeV$, down to the lower scale M^*. The outcome is that the evolution according to (21) lasts until reaching the scale M^*, and after that most of SUSY particles decouple, the inflationary solution becomes unstable such that the FLRW phase can start. In fact, the crucial point is the existence of a nonvanishing f as it eventually tempers stable inflation allowing favorable conditions for the universe to tilt into the FLRW phase [12, 5, 6].

As a result, we arrive at a consistent picture of the graceful exit from the anomaly-induced inflation to the FLRW stage. It is easy to see that this conclusion does not change if we choose another scale for the SUSY breaking. For a lower scale of SUSY breaking (e.g. $10^{10}\,GeV$ as in the Pati-Salam model, or even $1\,TeV$ as in the MSSM) there is no need to impose the constraint on the SUSY spectrum. In this case the area of applicability of our leading-log approximation is the same as the applicability of Eq. (21) and (much more important!) this approximation is valid until the SUSY breaking scale. The main difference will be that, for a lower M_{SUSY}, the total number of e-folds will increase dramatically, and that the inflation will consume more time. But, the evolution at the last stage of inflation will be quite similar as can be seen from Eq.(21). Another observation is that, in agreement with (11), only spinor fields contribute to the value of f. Therefore, as it was anticipated in the introduction, taking the masses of the fermion fields into account we arrive at a tempered form of inflation; besides, the Universe enters the phase of unstable inflation [5, 9] with such initial conditions that it ends up with the FLRW behavior. Furthermore, according to (11) the result (21) is universal, for it does not depend on the choice of the dilatation symmetry breaking scale M. If interpreted physically, one can put constraints on M using the macroscopic forces mediated by the field σ, demanding that this forces should have the submillimeter range, similarly as in [25].

Finally, for a really successful exit from the inflation phase we need to evaluate the dynamics of $H(t)$ during the last 65 e-folds of inflation. The importance of this calculation is related to the fact that the amplitude of the gravitational waves [7] is consistent with the observable range of anisotropy in the CMBR if, during the last 65 e-folds of the inflation, the Hubble constant H does not exceed

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[6] Notice that $|16\pi b| = \mathcal{O}(1)$ in the MSSM, and it is much larger than 1 in any typical SUSY GUT.

[7] Let us remind that the spectrum of the gravitational waves in the exponential phase of inflation is almost flat and agrees with all observational data [8].

$10^{-5} M_P$. This is because the fluctuations in the amplitude h of these waves is evaluated using $\delta h/h = H/M_P$ and, on the other hand, is related to the fluctuations in the temperature of the relic radiation. Thus, it has to satisfy the relation $\delta h/h = \delta T/T = \mathcal{O}(10^{-5})$. At the lowest end of the inflation interval this condition corresponds, e.g. in the SUSY-GUT case to a final scale value $H_f = M^* \lesssim 10^{14} GeV \approx 10^{-5} M_P$. We expect that after the onset of the approximate plateau in Fig. 1 (b), where the transition to an unstable phase occurs, the universe will take a while before entering the FLRW phase, i.e. the latter will actually initiate at some point well over the plateau. We have numerically checked that $H(t)$ decreases very fast on it. For instance, a 15% increase of the time at the beginning of the plateau amounts $H(t)$ to diminish two orders of magnitude [8]. So in general $H(t)$ will decrease further below M_{SUSY}, and the difference between $H_f = M^*$ and M_{SUSY} at the moment of the transition can be significant, say one or two orders of magnitude. Hence M_{SUSY} can be $10^{16} GeV$ and this does not create problems with CMBR. Next we have to derive the value H_i just some number of e-folds $n_e \gtrsim 65$ before the SUSY breaking point H_f, where as usual n_e is defined through $a_f/a_i = \exp[n_e]$. We obtain the following relations:

$$H_f^2 = H_1^2 + \frac{1}{48\pi^2 b} \sum N_i m_i^2 \ln a_f \,,$$

$$H_i^2 = H_1^2 + \frac{1}{48\pi^2 b} \sum N_i m_i^2 \ln a_i = H_f^2 - \frac{n_e}{48\pi^2 b} \sum N_i m_i^2 \,. \tag{23}$$

Notice that $H_i^2 > H_f^2$ because $b < 0$. However, if we suppose that $H_f = M^*$ and that the sum $\sum N_i m_i^2$ is of the order of M^{*2}, then H_i is of the same order of magnitude as H_f. In other words, the amplitude of the gravitational waves produced by the anomaly-induced inflation can be consistent with the magnitude of the CMBR. Remarkably, this result can be achieved without specifying the details of the gauge model. It is sufficient to make some reasonable suppositions about the mass spectrum of SUSY particles. The numerical analysis confirms the conclusion derived from the approximate formula (23). On the other hand the final conclusion regarding the consistency with the CMBR observations require an explicit derivation and analysis of the metric and density perturbations in the last 65 e-folds of inflation, between H_i and H_f. Such study is beyond the scope of the present considerations, and it may require an elaborated analysis of the time dynamics of the Hubble parameter (defining the scale μ of the gravitational interactions) in the given fundamental theory. Even at the level of our relatively simple effective framework, $H(t)$ is obtained only after numerically solving the non-linear fourth order differential eq.(17). However, in the gravitational wave sector, one can make some qualitative observations without explicit calculations. The corresponding analysis has been performed, for the case of constant H, in [11] and later on in [8] in the effective action framework when all parameter dependences become explicit. According to this work, the perturbation spectrum strongly depends on the parameters of the classical vacuum action and also on the choice of the quantum vacuum state of the induced theory (which was previously discussed in [26] in relation to the analysis of Hawking radiation from the black holes). The general conclusion is that the spectrum is very close to the Harrison-Zeldovich one for the sufficiently small value of the relevant vacuum parameter a_1 (consistent with the renormalization group) and with the most natural choice (in comparison to the black hole case) of the quantum vacuum. It is clear that the same possibilities of changing the perturbation spectrum exist for the non-constant H.

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[8]This can roughly be compared (as in the original model [1], though of course in a different sense) to the situation in a supercooled phase transition in which energy decreases a lot before the transition really takes place.

Moreover, since in the phenomenologically important period of inflation the scale factor changes by more than 65 e-folds while $H(t)$ remains to be the same order of magnitude, one can suppose that the constant-H terms will dominate in the equations for the metric perturbations and that the result will not be very different from the one of the constant H in Ref. [8]. Therefore, the anomaly-induced inflation has some predictive power in the description of the perturbations spectrum. But, the small details of this spectrum can be changed by adjusting the parameters of the classical vacuum action and the quantum vacuum. As a result, we may hope to fit with the present and future experimental data within this model. In principle, when the amount of such data will become sufficiently large, one can expect to achieve some additional information concerning the spectrum of the high-energy theory in this framework.

5. Conclusions

In summary, we have considered an effect of particle masses on the anomaly-induced inflation. The method of calculation was based on the local Cosmon Model [13], i.e. the conformal description of the massive fields, and on the conventional method of deriving the anomaly-induced effective action. The output of our approach agrees with the expressions expected from the renormalization group. The cosmological application of our result is that, independent of the details of the particle content of the model, the (spinor) matter fields slow down inflation. Together with the supersymmetry breaking effect [9], this provides the qualitative basis for the graceful exit from the stable anomaly-induced inflation. Furthermore, there is the possibility that under certain assumptions concerning the spectrum of the SUSY GUT, the amplitude of the gravitational waves is consistent with the CMBR constraints. The precise quantitative description will of course require to go into the details of a more fundamental theory (superstring theory or M-theory) underlying this effective approach. In the meantime we see that in the anomaly-induced model there are some indications to a phenomenologically consistent picture of inflation without introducing an *ad hoc* inflaton and without fine-tuning the parameters and/or the initial data.

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